CALCULATION OF THE ROOT DENSITY REQUIRED FOR GROWTH IN SOILS OF DIFFERENT P-STATUS

M. VAN NOORDWIJK and P. DE WILLIGEN

Institute for Soil Fertility, Haren (Gr.), The Netherlands

Summary

Root density determines the physical availability of mobile soil phosphorus to the plant. Calculations about diffusion of phosphorus to the roots were made, with the assumption that the rate of uptake is determined by the requirements of the crop in the linear growth phase. The poorer the soil, the higher must be the root density to be able to extract sufficient phosphorus from the soil. This relation is quantified analytically for a linear adsorption process and through numerical simulation for the curvilinear adsorption under agricultural conditions. An agricultural evaluation of plant-available soil phosphorus by extraction with water is possible, but the best volume ratio of soil to water to be used depends on the root density.

Introduction

A gap still exists between the experimental basis of fertilizer recommendations, and an explanation of the processes involved through knowledge of soil physical chemistry and plant behaviour. The explanations usually are only qualitative. The first bridges across this gap have only recently been build by quantitative models of solute movement in the soil-root system (Nye and Tinker, 1977). Our approach is a little more practical and applied than that of other models, starting with the phosphorus requirements of a crop at its maximum rate of growth, as determined by factors other than phosphorus shortage.

The main problem is to determine how extensive a root system must be to extract enough phosphorus to sustain this growth rate in soils of different P-status. The agricultural approach is the reverse: how high must the P-status of a soil be to give maximum growth of crops of different rooting densities. Closely related to this is the question how the desired P-status can best be characterized. Most literature on soil extraction methods deals with chemical aspects of extraction with water or any of a wide range of acids (Ansiaux, 1977), to simulate extraction by roots. But since phosphorus must diffuse to the roots and since this process is strongly affected by adsorptive phenomena in the soil, there are important physical problems too in characterizing the availability to roots. The distance a phosphate ion must traverse from its position in the soil to the nearest active root surface is an important parameter, which depends on root density (i.e. length of roots per unit volume of soil). Our approach is to simplify the problem very roughly, to obtain results that only indicate orders of magnitude, but which can be easily generalized. Our model is based on a number of simplifying assumptions.

Assumptions

Physical chemistry of phosphorus in the soil

We assume that only two fractions of phosphorus are important to the plant: inorganic phosphorus in the aqueous phase and that adsorbed to the solid phase, together forming the mobile phosphorus. The two fractions are related by an adsorption isotherm, which is a characteristic of a given soil. We assume this adsorption relation to be valid for the desorption too (i.e. no hysteresis) and that local equilibrium is maintained at all times. In the Netherlands phosphorus recommendations for arable crops are based on the Pw-value, the amount of phosphorus extracted with water, with a volume ratio of soil to water of 1:60 (Van der Paauw, 1971). This desorption can be approximately calculated for this and any other soil to water ratio from the adsorption isotherm (De Willigen and Van Noordwijk, 1978).

Geometry of the soil-root system

We assume a homogeneous distribution of roots of uniform thickness (radius 0.025 cm) in the phosphorus-containing upper layer of the soil (20 cm in our calculation). As we assume the uptake rate of all roots to be equal, an equivalent cylinder of soil is attributed to every root, without transport across its outer boundary. We do not consider mycorrhiza or root hairs and in our first approximation we assume a stationary root system without growth.

Uptake of phosphorus by the root

We assume the uptake rate by the roots to be regulated completely by the requirements of the plant, defined by growth and phosphorus content of the dry matter. This implies that the uptake of individual roots has to be correspondingly higher in a sparse root system than in a dense system. We assume that all roots have the same possibilities for uptake, independent of age (Clarkson *et al.*, 1977). The required uptake cannot be realized when the concentration at the root surface falls below some limiting concentration determined by the relation between concentration and uptake rate, with a maximum uptake rate (which determines the minimum amount of roots for sufficient uptake) of 0.5 10^{-12} Mol cm^{-2} s^{-1} and in the concentration dependent phase a root absorbing power of 2.10^{-4} cm s^{-1} (Nye, 1977). Thus far we only considered crops in the linear phase of growth with a growth rate of 200 kg ha^{-1} day^{-1} (Sibma, 1968) and a constant P-content of 2.2 mg P g^{-1} dry matter.

We express the possibilities for uptake of a soil-root system by t_c, the time during which a root system may realize its required uptake rate. After t_c the plant cannot keep its phosphorus content at the most favourable level and production will be affected. Increasing the root density would of course be the best strategy for the plant, but this diminishes the dry matter available for shoot growth.

Analytical Solution in Case of a Linear Adsorption Isotherm

For circumstances where the adsorption isotherm is approximately linear, an analytical solution for the concentration profile around the root could be derived. The variables are combined into the following dimensionless groups:

u = concentration (actual concentration divided by the original)

x = distance to the centre of the root (divided by Ro, the radius of the root)

ρ = radius of soil cylinder (divided by Ro)

τ = time (multiplied by "apparent diffusivity" - the adsorption reduces the diffusivity in water by a factor of 10 to 2000 - and divided by Ro^2)

ϕ = demand-supply relation (apparent diffusivity times amount of mobile phosphorus initially present divided by uptake per unit surface area of soil per day times Ro)

η = depth of P-concentration zone (divided by Ro).

The solution for the concentration in space and time is:

$$(1)\quad u(x,\tau) = 1 + \frac{\rho^2}{2\eta\phi}\left[\frac{2\tau}{1-\rho^2} + \frac{x^2-\rho^2}{2(1-\rho^2)} + \frac{\rho^2}{1-\rho^2}\ln\left(\frac{\rho}{x}\right) - \frac{\rho^2}{(1-\rho^2)^2}\ln\rho - \frac{\rho^2+1}{4(1-\rho^2)} - \pi\sum_{n=1}^{\infty}\frac{e^{-\alpha_n^2\tau}}{\alpha_n}\,\frac{J1(\rho\alpha_n)\,\{J0(x\alpha_n)\,Y1(\alpha_n) - Y0(x\alpha_n)\,J1(\alpha_n)\}}{J1^2(\alpha_n) - J1^2(\rho\alpha_n)}\right]$$

where J0, Y0, J1 and Y1 are Bessel functions of the first

(J) and second (Y) kind and of order 0 and 1. The α_n are the roots of $J1(\alpha)\,Y1(\rho\alpha) - J1(\rho\alpha)\,Y1(\alpha) = 0$. The series part of this solution was given by Carslaw and Jaeger (1959) in section 13.4.

The first part of this formula does not contain products of distance and time, while the infinite sum does. This means that if the second part can be neglected, the depletion of the soil has the same rate at every distance from the root and is constant in time, i.e. is a steady rate process. After some time, determined by the demand-supply relation and root density, the second part may in fact be neglected as time appears in the negative exponential term only. The depletion converges to a steady rate as shown in Figure 1.

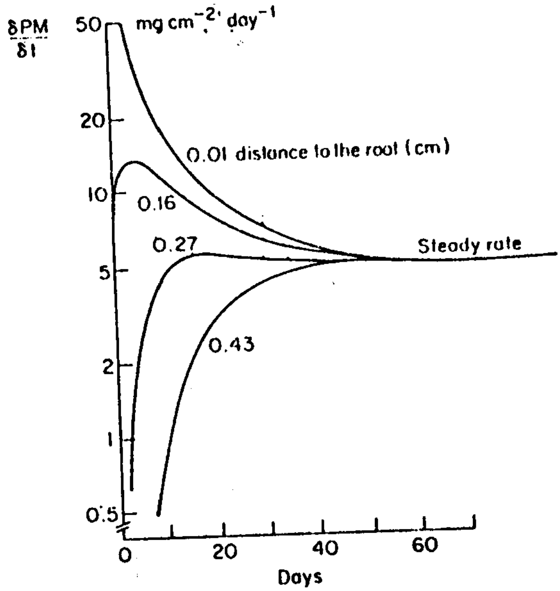

Fig. 1. Depletion of the mobile phosphorus (Pm) in the soil at different distances from the root surface. After some time the rate of depletion becomes independent of the distance ("steady rate").

The time t_c can be found by solving for the concentration at the root surface equal to the limiting concentration. When the steady rate is reached it is a simple matter to solve explicitly:

$$(2)\quad \tau_c = \phi' n \frac{(\rho^2-1)}{\rho^2} - \left(\frac{\rho^2}{2} \ln\rho - \frac{3\rho^2}{8} - \frac{1}{8\rho^2} + \frac{1}{2}\right)\left(\frac{\rho^2}{\rho^2-1}\right)$$

with ϕ' differing from ϕ in that the amount of phosphorus at the limiting concentration is substracted from the amount of mobile phosphorus at t_o.

Solution by Simulation in the Case of a Non-linear Adsorption Isotherm

The adsorption isotherms, determined for five different soils, are distinctly non-linear in the agriculturally important concentration range. They may be described by two Langmuir terms (Holford and Mattingley, 1976a). No analytical solution could be found, so a computer simulation model was used, tested with the linear adsorption solution. For the five soils at three different Pw-values (mg P_2O_5 dm^{-3} soil), viz., 10, 30 and 50 (agricultural evaluation poor, sufficient and high) t_c was calculated (for details: De Willigen and Van Noordwijk, 1978). The results in Figure 2 show that the poorer the soil, the higher the root density has to be for good growth. An agricultural norm of t_c = 100 days, may be reached by a root density of 1 cm^{-2} at Pw 50, a root density 2 to 3 at Pw 30 and a much higher density, depending on the soil, for Pw 10. We conclude that the value of Pw is a good measure of possibilities for uptake in a certain range, but for higher root densities the Pw value does not have the same meaning for different soils.

The diameter of roots and the presence of root hairs likewise are important, as may be seen in Figure 3. With thinner roots a larger root density is required for the same uptake. But the same biomass per unit volume of soil is more efficient when distributed over a larger number of thinner roots. Provided that the distribution of the roots stays homogeneous and the physiological abilities for uptake and transport along the root remain constant, there is no optimum root diameter for uptake, as the thinnest are best.

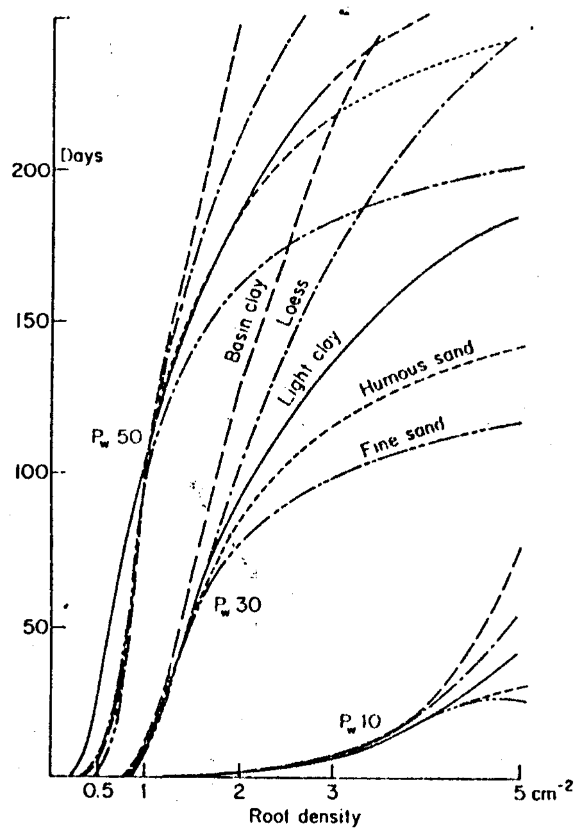

Fig. 2. Simulation results with a non-linear adsorption isotherm. The time during which the uptake is sufficient for plant requirements (t_c) is given as a function of root density (cm of roots cm^{-3} of soil) and P-status.

Some remarks can be made on Pw as a method of soil analysis. The optimum soil/water ratio to be used in this method was derived by means of pot experiments with young potato plants (Van der Paauw, 1971). Our results suggest that the optimum ratio of soil to water for characterizing availability depends on the root density, as shown in Figure 4. Different values of Pw(v) according to the volume ratio (v) water to soil were calculated from the adsorption isotherms and compared with t_c for our model calculations and actual uptake data of pot experiments of

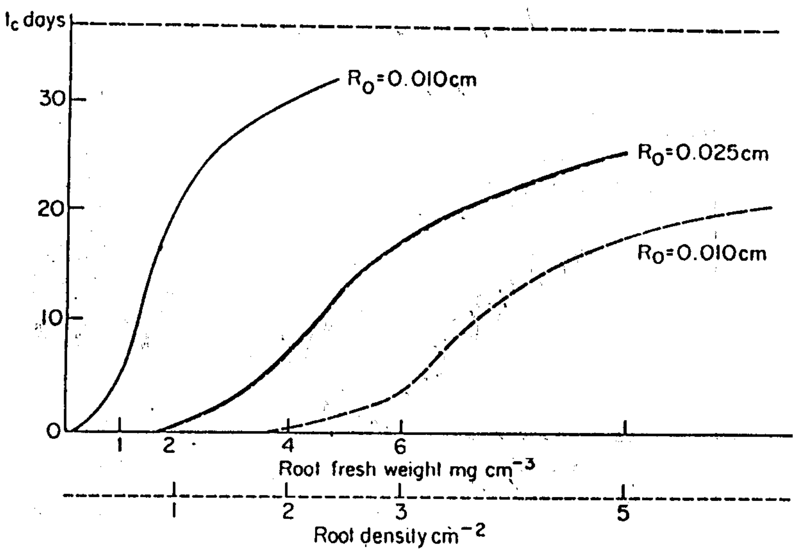

Fig. 3. Effect of root diameter on possibilities for uptake. For the fine sand of Pw 10 of Figure 2, a smaller root diameter is compared both on the basis of root density and on the basis of root biomass.

Holford and Mattingly (1976b) with *Lolium perenne* (with unknown, but presumably high root density estimated to be over 10 cm^{-2}, Mattingly pers. comm.). The higher the root density, the larger the fraction of mobile phosphorus ("chemically available") that can be transported to the root at the required rate ("physically available"). To simulate this in soils with different adsorption capacities more water has to be used in the extraction procedure. The agricultural experience that Pw (60) is a good measure for arable crops but much worse for permanent grassland with a much higher root density may well be interpreted along these lines.

Acknowledgements

Thanks are due to Dr. H.A. Sissingh for measuring the adsorption isotherms and to Dr. N.N. Temme (Mathematisch Centrum, Amsterdam) and Dr. P.A.C. Raats for mathematical help.

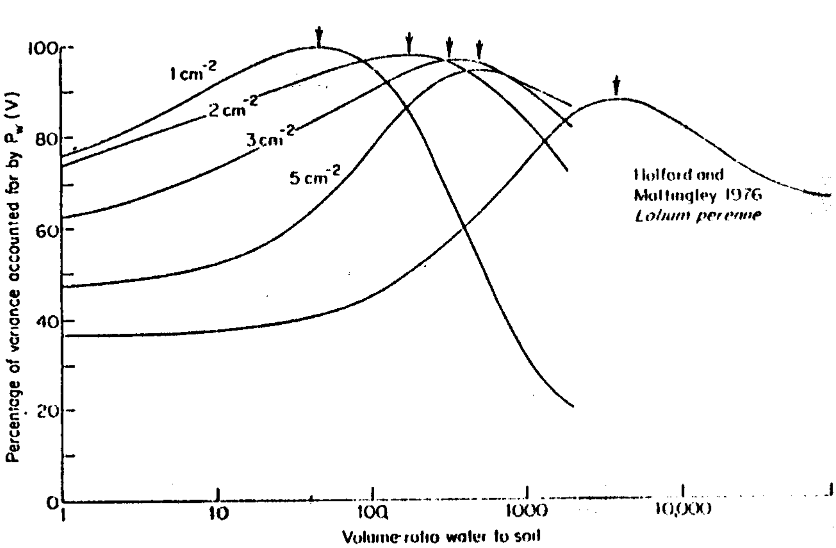

Fig. 4. Percentage of the variance in plant performance on different soils accounted for by Pw(v), i.e. calculated values for a water extract of the soil at different volume ratios of soil to water (v). The upper four lines represent the correlation of t_c (the length of the period of undisturbed uptake, calculated in our model for different soils and P-status) with calculated values of Pw(v), for each root density. In the lower line the actual uptake of phosphorus after 40 days by *Lolium perenne* in pot experiments on 3 calcareous soils, was correlated with Pw(v) values calculated from adsorption isotherms (data of Holford and Mattingly, 1976). The optimal extraction ratio increases with root density.

References

Ansiaux, J.R. (1977). The level of soil phosphorus status. *Phosphorus in Agriculture*, 70, 1.

Carslaw, H.S. and Jaeger, J.C. (1959). "Conduction of Heat in Solids", 2nd Edition. Clarendon, Oxford, 510 pp.

Clarkson, D.T., Sanderson, J. and Scattergood, C.B. (1977). The modification of phosphate transport of barley by phosphorus deficiency. *Agric. Res. C. Letcombe Lab. Ann. Rep. 1976*, 9.

Holford, I.C.R. and Mattingly, G.E.G. (1976a). A model for the behaviour of labile phosphate in soil. *Plant and Soil*, 44, 219.

Holford, I.C.R. and Mattingly, G.E.G. (1976b). Phosphate adsorption and availability [to the] plant of phosphate. *Plant and Soil*, 44, 377.

Nye, P.H. and Tinker, P.B. (1977). "Solute Movement in the Soil-Root